PARAMETERS OF A HYDROGEN PLASMA IN A SUPERSONIC SPHERICAL SOURCE

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The system of equations for a steady-state source of a nonequilibrium hydrogen plasma have been integrated numerically. Collisional-radiative recombination is considered the limiting elementary step. The calculation was carried out for plasmas optically thin and optically thick for all types of radiation and for a plasma optically thick only for reasonance radiation, in order to determine the effects of reabsorption. There is considerable disruption of the thermal and ionization equilibrium in the flow region. Reabsorption of the radiation leads to a large discrepancy between the electrons and heavy-particle temperatures and to a decrease in the recombination rate. The population of the hydrogen states is determined in a quasi-steady-state approximation from the electron concentration and temperatures. The radiation gain is calculated for several transitions.

During rapid expansion of a plasma, such as occurs in supersonic escape into a vacuum, the gas is in a greatly nonuniform state. Typical of this state are a difference between the electron and heavy-particle temperatures ($T_e > T$), an electron concentration large in comparison with the equilibrium concentration, N_e, and a surplus of atoms in the higher excited states. Under certain conditions a population inversion of the excited states occurs, which may be of considerable practical interest.

These phenomena have been studied theoretically and experimentally in [1-6]. In this study, we have calculated the plasma parameters, including the state populations, for flow in a spherical source. The working medium was taken as hydrogen, for which the parameter values entering the calculation are comparatively well known.

Basic Assumptions. We assume that the hydrogen plasma consists of electrons, ions, and atoms at the initial spherical surface. With the initial parameters chosen (see below), the initial degree of dissociation is large, and molecule formation in the flow can be neglected, since the recombination rate is small in comparison with the expansion rate. The equilibrium concentration of negative hydrogen ions under the conditions of interest here does not exceed 10^{-5} [7].

We consider the flow after disruption of the ionization equilibrium. We assume the velocities of all components are equal and that the plasma is quasineutral.

We assume collisional-radiative recombination to be the limiting elementary step; this is the successive recombination of electrons at the higher excited states in triple collisions and cascade transition to the ground state (see, e.g., [8]). In order to determine the state-population kinetics for hydrogen as the plasma moves, we must solve jointly the differential population-balance and hydrodynamical equations.

When, however, the concentration of excited atoms, N_k ($k \neq 1$), satisfies the conditions

$$N_k \ll N_1, \quad N_k \ll N_e, \quad (k \neq 1), \tag{1}$$

a quasistationary solution of the population-balance system is possible. The relaxation time for a level $k \neq 1$ is considerably smaller than the ground-state relaxation time or that of free electrons. The population

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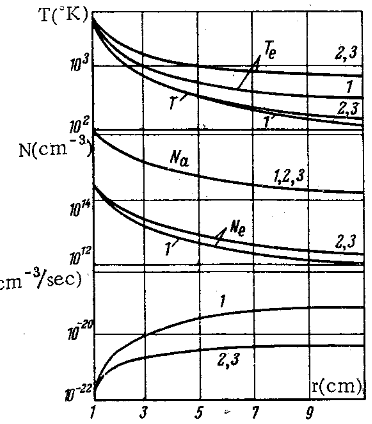

Fig. 1. Recombination coefficient, temperature, and particle concentration in the supersonic source. 1) Plasma optically thin for all types of radiation; 2) optically thin for all types of radiation except for the Lα line; 3) optically thick for all types of radiation.

of a state with $k \neq 1$ changes extremely slowly with respect to the rate of formation and destruction of systems in this state. The differential population-balance equations are replaced by a system of algebraic equations, permitting determination of N_k ($k \neq 1$) as functions of N_e and T_e, which are known from the hydrodynamical calculation. In addition to condition (1), the following must be satisfied:

$$\tau_k \ll \tau_h, \tag{2}$$

where τ_k is the relaxation time for a state with $k \neq 1$, and τ_h is the characteristic hydrodynamical time.

The parameter ranges considered below conform to conditions (1) and (2). Far from the source pole, at $T_e \lesssim 500°$ K, these conditions are violated, and the quasistationary approximation can be used only for semiquantitative results.

The System of Equations. The system of hydrodynamical equations, including those for conservation of mass, momentum, and energy, and the equation of state, are:

$$\frac{d}{dr}(N_e ur^2) = -\alpha N_e^2 r^2, \tag{3}$$

$$\frac{d}{dr}(N_a ur^2) = \alpha N_e^2 r^2, \tag{4}$$

$$dp_e / dr = -eN_e E, \tag{5}$$

$$m_a(N_e + N_a) u \frac{du}{dr} = -\frac{d(p_i + p_a)}{dr} + eN_e E, \tag{6}$$

$$\frac{d}{dr}\left(N_e ur^2 \frac{5}{2} kT_e\right) = Qr^2 - I^* \frac{d}{dr}(N_e ur^2) - N_e ur^2 eE, \tag{7}$$

$$\frac{d}{dr}\left[N_e ur^2\left(\frac{5}{2}kT + \frac{m_a u^2}{2}\right) + N_a ur^2\left(\frac{5}{2}kT + \frac{m_a u^2}{2}\right)\right] = -Qr^2 + N_e ur^2 eE, \tag{8}$$

$$p_e = N_e kT_e, \tag{9}$$

$$p_i = N_e kT, \tag{10}$$

$$p_a = N_a kT, \tag{11}$$

$$Q = \frac{N_e^2 e^4}{(4\pi\varepsilon)^2 m_a}\left(\frac{8\pi m_e}{kT_e}\right)^{1/2}\left(\frac{T}{T_e} - 1\right)\ln\left[\frac{72\pi^2 (kT_e)^3 \varepsilon^3}{N_e e^6}\right].$$

Here p is the pressure, N is the concentration, u is the velocity of directed motion, m is the particle mass, T is the temperature, and r is the distance from the source pole. The subscripts e, i, and a refer, respectively, to electrons, ions, and atoms. The values of the recombination coefficient, $\alpha(N_e, T_e)$, were taken from [8].

The rate Q of energy transfer from electrons to heavy particles during elastic collisions, when there is a high degree of ionization, is governed completely by electron-ion collisions. The quantity I^* is the part of recombination energy which is returned to the electron gas during deexcitation of the excited states by the second type of electron collisions. Part of the recombination energy, $I - I^*$, goes into the special-line radiation. Some part of this energy can also go into the thermal energy of the electrons if part of the radiation is reabsorbed in the plasma. Reabsorption of resonance radiation is the most important energetically.

To determine the effects of reabsorption on the plasma parameters, we consider three cases.

1. A plasma which is optically thin for all types of radiation [9]:

$$I^* = 3.1 \cdot 10^{-4} N_e^{1/6} T_e^{1/12} I.$$

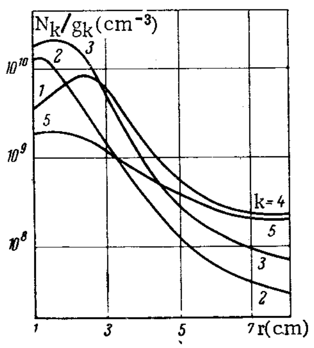

Fig. 2

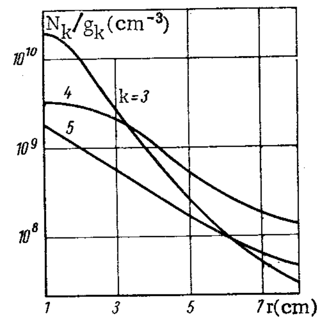

Fig. 3

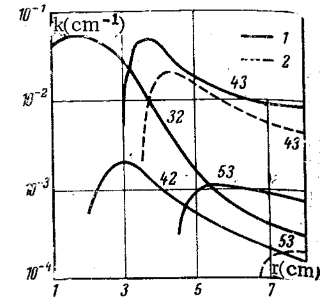

Fig. 4

Fig. 2. Population of states in a source optically thin for all types of radiation.

Fig. 3. Population of states in a source optically thin for all types of radiation except the L_α line.

Fig. 4. Variation of the gain along the source radius.

2. A plasma which is optically thin for all types of radiation except for the L_α line:

$$I^* = 3.1\cdot 10^{-4} N_e^{1/6} T^{1/12} I + I_{21} (I_{21} = 10.15\,\text{eV}).$$

3. A plasma which is optically thick for all types of radiation:

$$I^* = I \quad (I = 13.53\,\text{eV}).$$

Plasma Parameters. By means of simple transformations, the system (3)-(11) was reduced to a system of five dimensionless first-degree equations suitable for machine solution. The numerical integration was carried out on the "Ural-1" computer by the Runge-Kutta method with a variable step. The following initial conditions were chosen: $r_0 = 1$ cm, $N_{e0} = N_{i0} = 10^{15}$ cm^{-3}, $N_{\alpha 0} = 10^{17}$ cm^{-3}, $T_{e0} = 5\cdot 10^3$°K, $T_0 = 4.5\cdot 10^3$°K and $u_0 = 10^6$ cm/sec.

The initial values of T_e and T were chosen from the balance conditions:

$$Q = I^* \alpha N_e^2.$$

Figure 1 shows the calculated values of T_e, T, N_e, N_α and α for the flow region $1 \le r \le 10$ cm. The parameter values for the optically thick plasma (case 3 in Fig. 1) are practically the same as for the plasma which is optically thick only for the L_α line (case 2). A comparison of the results for cases 1, 2, and 3 shows that reabsorption of the radiation causes increased T_e, N_e, and degree of ionization, as well as a decreased recombination coefficient. The latter circumstance is due to the strong temperature dependence, $\alpha \sim T^{-9/2}$. At large values of r, the recombination coefficient changes slowly. The sharp increase in α near the initial surface is related to the value adopted for the electron temperature T_e, which, generally speaking, is governed by the prehistory of the flow. In an actual expansion of a plasma, smaller gradients of α should be expected at $r \simeq 1$ cm.

Concentrations of Excited Atoms. The concentrations of hydrogen atoms in various excited states were calculated as functions of T_e and N_e in [8, 10, 11]. Figures 2 and 3 show the populations N_k of several of the lower excited states, adjusted for statistical weights g_k, as calculated from the data of [8] and from our values of T_e and N_e for various values of r.

The higher excited states ($k \ge 5-6$) are in equilibrium with each other and with the electron continuum. In the case of an optically thin plasma, there is a population inversion of the higher states with respect to the second state over almost the whole flow field; starting with $r \simeq 3-4$ cm, this is also true with respect to the third state. For a plasma which is optically thick for the L_α line, the population of the second state increases to $10^{12}-10^{13}$ cm^{-3} (see below). Therefore, we cannot discuss inversion with respect to the second state in a weakly ionized plasma with a large optical thickness. Reabsorption of the L_α line has practically no effect on the populations N_3 and N_4 if L_α does not exceed 2000 and 4000°K, respectively. Starting at $r \simeq 3$ cm, inversion of the pair of states 4-3 occurs, and at $r \simeq 7$ cm, of the pair 5-3.

Radiation Gain and Effect of Reabsorption. The radiation gain with a population inversion is calculated from

$$K_{kl} = \frac{1}{4\pi^2} s(\lambda) \lambda_{kl}^2 g_k A_{kl} \left(\frac{N_k}{g_k} - \frac{N_l}{g_l}\right). \tag{12}$$

For a Doppler contour, we have $s(\lambda) = \sqrt{\pi \ln 2}\, \lambda^2 / c\Delta\lambda$.

For a Stark contour, we have

$$(\lambda) = \lambda^2 / \pi c \Delta\lambda,$$

where $s(\lambda)$ is the shape factor for the spectral line, and A_{kl} is the probability for a spontaneous transition.

For the source considered here, the line halfwidth $\Delta\lambda$ is in the range 0.2-2.5 Å; the contour of the H_α line is governed practically completely by the Doppler effect; for transitions from the fourth and fifth states at $r \lessdot 2-3$ cm, Stark broadening dominates.

Figure 4 shows the gains for an optically thin plasma and for a plasma which reabsorbs only the L_α line. The gain is quite large for the 4-3 ($\lambda = 1.88\,\mu$) and 3-2 ($\lambda = 0.65\,\mu$) transitions in the case of an optically thin plasma. A gain of the order of 3% is achieved with a radiating-layer thickness of 1-3 cm, sufficient to compensate the end losses in the resonator. There is also a large gain for the 4-2 (H_β) transition, and for the 4-3 transition in the plasma which is optically thick for the L_α line.

Reabsorption of radiation, which occurs during transitions to the second state, has little effect on the values of N_e and T_e, but it can significantly reduce the inversion.

We can estimate the thickness of the plasma source for the L_α and H_α lines. Significant reabsorption of lines with a Doppler contour occurs in a layer of thickness x when

$$k(L_\alpha) = 6\cdot 10^{-12} T_e^{-1/2} N_1 x > 1, \quad k(H_\alpha) = 5\cdot 10^{-11} T_e^{-1/2} N_2 x > 1.$$

Assuming that the radiation leaves the source in the radial direction, we find, for a point at r, the condition

$$k(L_\alpha) \simeq 10^{-13} N_1(r) r > 1, \quad k(H_\alpha) \simeq 10^{-12} N_2(r) r > 1.$$

A numerical analysis shows that $k(L_\alpha) \gg 1$ over the whole field; the resonance radiation is completely locked in; and an optically thin plasma is not achieved. The rate at which the second state is populated can be found from

$$dN_2 / dt = \alpha N_e^2 - R N_e N_2$$

(the values of the factor R are given in [8]). At a distance of the order of 0.1 cm from the initial surface, the quantity N_2 reaches a value of 10^{13} cm^{-3}, and then falls off rather slowly. Accordingly, we have $k(H_\alpha) > 1$; the H_α line is reabsorbed in much of the source; and the population N_3 may grow considerably.

A more accurate determination of the concentrations of excited atoms, the inversion, and the gain requires solution of the problem incorporating radiation transport in the source.

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EFFECT OF A TRANSVERSE MAGNETIC FIELD ON THE HEAT FLUX TO THE ELECTRODES OF A SHARPLY-DEFINED DISCHARGE

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Results are presented for measurements of heat flux to "point" electrodes in crossed E-H fields. The magnetic field strength reached a value of 10,000 Oersted and the current a value of 10 kA. The method of [1] was used in the measurements. The results obtained are discussed.

It is known [2-4] that the potential drop, V, at the electrodes of an MHD system may increase with increase in magnetic induction. In [5], a linear dependence of V on $\omega\tau$ (product of the electron Larmor frequency and the time between electron collisions with atoms) was obtained. It was shown in [4] that, for $\omega\tau = 0.7-2.5$, the primary potential losses in a small MHD channel occurred at the electrodes and, under the conditions of the experiment, constituted $1/3-2/3$ of the electromotive force.

If it is assumed that the heat flux to an electrode is equal to the product of the current and the potential drop at the electrode, then it should be expected that heat loss will increase with increase in magnetic induction. However, magnetization does not permit electrons to acquire the entire energy corresponding to the potential drop at the electrode. Besides, changes in gas density may occur in the cathode and anode regions due to the Hall effect [6], and electrode erosion may occur, which further complicates the picture of the phenomenon. Therefore, measurement of the heat flux to the electrodes in a magnetic field is useful not only for determining losses but also for clarification of the mechanisms involved in the processes in the electrode sheath.

Measurements of heat flux to electrodes without an external magnetic field at discharge currents to 10 kA are described in [1]. In further tests measurements were made of heat flux in a magnetic field. Some of the results obtained are presented below.

EXPERIMENTAL APPARATUS AND RESULTS

The experiments were conducted with a pulsed system. Pre-ionized argon was admitted to a channel with a 30×40 mm cross section. Electrodes having a diameter of 3.5 mm were placed in two opposing walls of the channel and were connected to a condenser having a capacitance of 150 μF. Detailed description of this device is given in [1].

The magnetic field was generated by a pair of Helmholtz coils. The coils of the magnetic system were supplied by means of discharge of a condenser bank having a capacitance of 900 μF. The time constant for current in the magnetic system (half period of 3 msec) was significantly greater than the time constant for discharge current in the channel (100 μsec) such that the magnetic field could be treated as constant during the discharge. Nonuniformity in magnetic field in the working portion of the channel over a length of 8 cm did not exceed 10% along the axis and 2% transverse to the channel. The magnetic field strength in the experiments reached 10,000 Oersted.

Synchronization of the instant of magnetic field onset and plasma admittance was accomplished by means of a spark gap and control unit which formed a high intensity impulse with regulated time delay.

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