Recombination in common envelope evolution

7th June 2018

ABSTRACT

Key words:

1 INTRODUCTION

These notes are adapted from Adam's version of January 2018. See also email thread from Jan 13-19. Relevant references:

- Lucy (1967)
- Roxburgh (1967)
- Paczyński & Ziółkowski (1968)
- Iben & Livio (1993) Sect. 3.5.2
- Han et al. (1994)
- Wagenhuber & Weiss (1994)
- Han et al. (1995)
- Han et al. (2002)
- Soker & Harpaz (2003)
- Harpaz (1998)
- Webbink (2008) Sect. 5 and 6
- Passy et al. (2012) Sect. 5.2.2
- Ivanova et al. (2013a) Sect. 3.3.2
- Ivanova et al. (2013b)
- Nandez et al. (2014)
- Ivanova et al. (2015)
- Ivanova & Nandez (2016)
- Kruckow et al. (2016)
- Sabach et al. (2017)
- Ivanova (2017)
- Nandez & Ivanova (2016)
- Grichener et al. (2018)
- Soker (2017)
- Clayton et al. (2017)
- Ivanova (2018)
- Soker et al. (2018)

The goal is to set up and solve a system of time-independent equations to model a steady flow which begins fully ionized but cools to the point where recombination occurs, releasing latent energy and driving the flow to become unbound.

2 EQUATIONS

We begin with the Euler equations in spherical symmetry (r-component of flow only). We are interested in a gas of hydrogen only and will track ionization via the ionization fraction x defined as

$$x = \frac{n_+}{n_0 + n_+}, \tag{1}$$

where n_0 and n_+ are the number densities of neutrals and ions, respectively.

The pressure is determined from the ideal equation of state,

$$P = \frac{\rho k_B T}{\mu}, \tag{2}$$

with T the temperature and with the mean molecular mass given by

$$\mu = \frac{m_H}{1 + x}, \tag{3}$$

with m_H the mass of a hydrogen atom. Plugging equation (3) into equation (2) we obtain

$$P = \frac{(1 + x)\rho k_B T}{m_H}. \tag{4}$$

The three flow equations are (e.g. Toro 2009, equation (1.107)).

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho u) = 0, \tag{5}$$

$$\frac{1}{r^2}\frac{d}{dr}(r^2 \rho u^2) + \frac{dP}{dr} = 0 \tag{6}$$

and

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 u(E + P)\right] = 0, \tag{7}$$

where ρ is density, u is radial velocity, P is pressure and E is the total energy density which includes kinetic and internal contributions,

$$E = \frac{1}{2}\rho u^2 + E_t. \tag{8}$$

The internal energy E_t contains a term corresponding to the translational kinetic energy of a monatomic gas, as well as a term that accounts for the electrostatic potential energy of the interaction (see notes by Gary Glatzmeir of UCSC (https://websites.pmc.ucsc.edu/~glatz/astr_112/lectures/notes6.pdf) available here),

$$E_t = \frac{3}{2}P + \frac{\chi \rho x}{m_H}, \tag{9}$$

where $\chi = 13.6\,\mathrm{eV}$ or 2.2×10^{-11} erg is the ionization energy of hydrogen. Substituting equation (9) into equation (8) and then the latter into equation (7), we obtain

$$\frac{1}{r^2}\frac{d}{dr}\left[r^2 u\left(\frac{1}{2}\rho u^2 + \frac{5}{2}P + \frac{\chi \rho x}{m_H}\right)\right] = 0. \tag{10}$$

Note that we have included the ionization potential energy not as a source term but as "latent" energy. This seems to be in keeping with the form that Ivanova et al. use. Note that we are assuming that all latent energy released is directly absorbed as heat. Therefore we do not need to track radiation.

Note that we have not included gravity in the first form of the system. We want to see how the release of ionization energy behaves before we see how it changes the binding of the gas. This means we are first interested in seeing how the released energy of ionization accelerates a flow without gravity.

To track the ionization fraction we have an additional advection equation for the continuity of the ionized species,

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left[r^2\left(\frac{x\rho}{m_\mathrm{H}}\right)u\right] = -\left(\frac{x\rho}{m_\mathrm{H}}\right)^2\beta(T), \tag{11}$$

where $\beta(T)$ is the recombination coefficient, which is inversely related to T (that is proportional to T raised to some negative power) and has dimensions $[\mathrm{L}]^3[\mathrm{T}]^{-1}$. The factor $(x\rho/m_\mathrm{H})^2 = n_+^2$ comes from the fact that the recombination rate is proportional to n_+n_e, where $n_\mathrm{e} = n_+$ is the electron number density. Equation (11) can be simplified using equation (5) to give

$$u\frac{\mathrm{d}x}{\mathrm{d}r} = -\left(\frac{\rho}{m_\mathrm{H}}\right)x^2\beta(T). \tag{12}$$

Alternatively, we can use the Saha equation to derive the following equation for x (https://websites.pmc.ucsc.edu/~glatz/astr_112/lectures/notes6.pdf):

$$\frac{x^2}{1-x^2} = \frac{2}{h^3 Z_0}\frac{(2\pi m_\mathrm{e})^{3/2}(k_\mathrm{B}T)^{5/2}\mathrm{e}^{-\chi/k_\mathrm{B}T}}{P}, \tag{13}$$

with the partition function of neutral hydrogen Z_0 being almost independent of T. By using equation (13) along with equations (2) and (3), one can obtain an expression for the adiabatic index $\gamma = (\ln P/\ln\rho)_\mathrm{ad}$ (https://websites.pmc.ucsc.edu/~glatz/astr_112/lectures/notes6.pdf),

$$\gamma = \frac{5+\left(\frac{5}{2}+\frac{\chi}{k_\mathrm{B}T}\right)^2 x(1-x)}{3+\left[\frac{3}{2}+\left(\frac{3}{2}+\frac{\chi}{k_\mathrm{B}T}\right)^2\right]x(1-x)}. \tag{14}$$

Our set of equations is given by:

- differential equation (5) (ρ, u),
- differential equation (6) (ρ, u, P),
- differential equation (10) (ρ, u, P, x),
- modified ideal gas law (4) (ρ, P, x, T),
- differential equation (11) (ρ, u, x, T) or result from Saha equation (13) (P, x, T).

So we have 5 equations in 5 unknowns; three or four of these are ordinary nonlinear differential equations in r. The temperature can be eliminated using the ideal gas law (4), and one can then solve the remaining equations for the variables $\rho(r)$, $u(r)$, $P(r)$ and $x(r)$, given boundary conditions for each of these four variables at $r = 0$ and $r \to \infty$.

Once a solution is obtained, it should be checked that the solution satisfies the constraint (14).

3 METHODS

3.1 Setup

Plan of execution:

4 RESULTS

5 DISCUSSION

6 CONCLUSIONS

ACKNOWLEDGEMENTS

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