General equation of state Riemann solvers: Convergence

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1 Introduction

Convergence is a critical concern in computational physics. It reflects how accurate can an algorithm be. In this short article, we will prove that the novel general equation of state Riemann solver does well in this aspect.

2 Ideal gas case: does the exact general EOS Riemann solver converge to the correct solution?

We compare the solution of exact general EOS Riemann solver with tabulated perfect gas EOS (constant γ) to the solution of exact ideal gas Riemann solver (constant γ). We carry out three shock tube (figure 1, 2, and 3) tests with different γ as the examples. Their left and right state is listed in table 1. It is shown that the exact general EOS Riemann solver can reduce to the ideal gas Riemann solver in perfect gas case.

ρ_L(g·cm^{-3})	v_L(km·s^{-1})	T_L(K)	ρ_R(g·cm^{-3})	v_R(km·s^{-1})	T_R(K)
10^{-13}	0	3000	10^{-15}	0	300

Table 1: Ideal gas Riemann problem initial left and right states. $dt = 0.25s$.

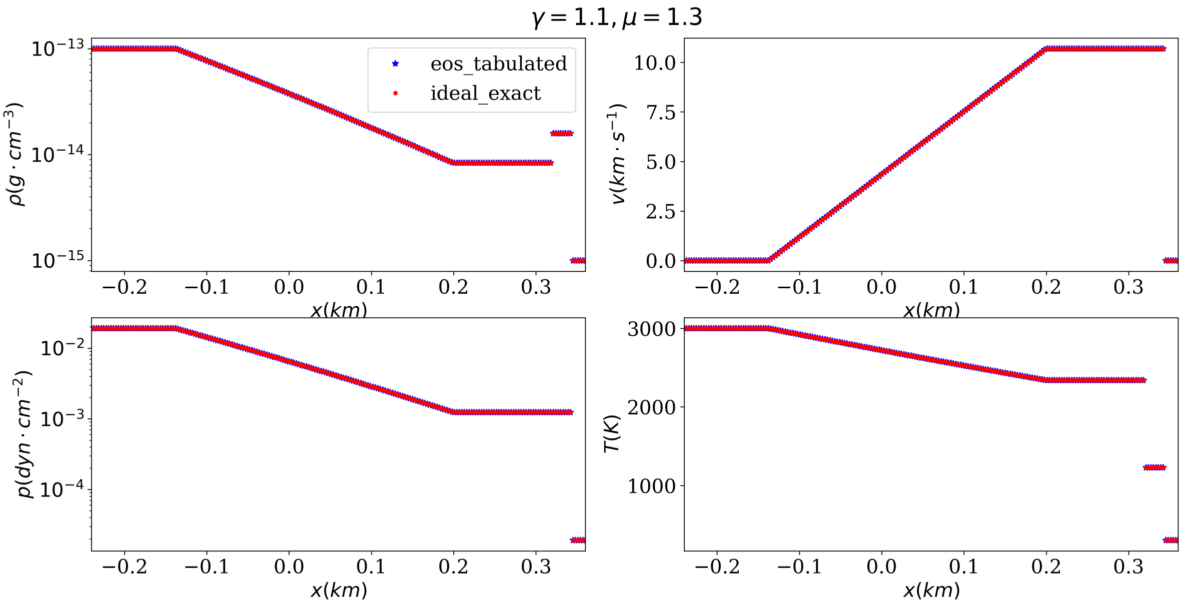

Figure 1

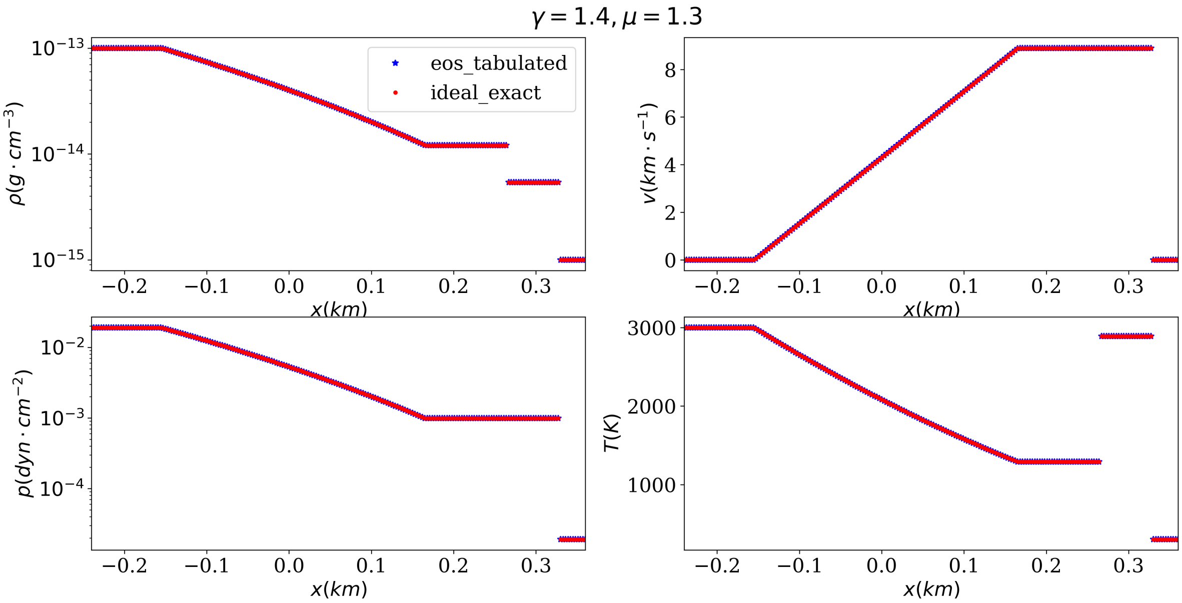

Figure 2

3 Real gas case: does the general EOS HLLC Riemann solver converge to the correct solution?

The answer is probably yes. We will show that the general EOS HLLC Riemann solver converge to the exact general EOS Riemann solver as the L1 norm of their difference approaches zero monotonically with increasing resolution. Explicitly, we calculate,

$$\delta_N = \int_{xmin}^{xmax} \|f_N(x) - f_{exact}(x)\| dx \tag{1}$$

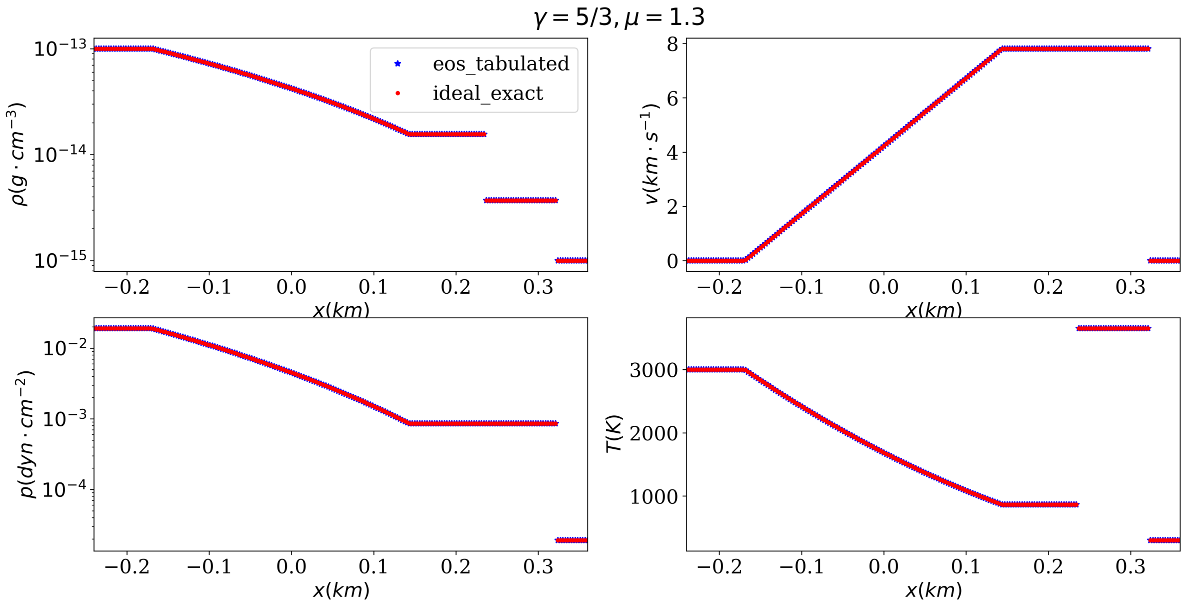

Figure 3

where f_N is the solution of any specific field, N is the number of points. f_{exact} is the solution of exact general EOS Riemann solver of any specific field. We choose $N = 100, 200, 400, 800$ and density field. We carry out three Riemann problem tests. Their left and right states are listed in table 2. The density plot of these three tests are shown in figure 4, 5, and 6. We list the δ_N of the three Riemann problem test in table 3.

test No.	ρ_L(g·cm^{-3})	v_L(km·s^{-1})	T_L(K)	ρ_R(g·cm^{-3})	v_R(km·s^{-1})	T_R(K)
test1	10^{-13}	0	3000	10^{-15}	0	300
test2	2×10^{-15}	20	300	10^{-15}	-20	300
test3	10^{-11}	-20	2000	10^{-11}	20	2000

Table 2: Real gas Riemann problem initial left and right states. $dt_{test1} = 0.02s$, $dt_{test2} = 0.05s$ *and* $dt_{test3} = 0.01s$.

test No.	δ_{100}	δ_{200}	δ_{400}	δ_{800}
test1(g·cm^{-2})	8.74×10^{-11}	6.14×10^{-11}	4.13×10^{-11}	2.76×10^{-11}
test2(g·cm^{-2})	1.26×10^{-10}	6.56×10^{-11}	4.75×10^{-11}	3.64×10^{-11}
test3(g·cm^{-2})	3.50×10^{-8}	2.22×10^{-8}	1.46×10^{-8}	9.77×10^{-9}

Table 3: For each row, the L1 norm δ decreases with increasing N (from left to right). Indicating that the general EOS HLLC solution is converging to general EOS exact solution.